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Computational Evaluation of a Latent Heat Energy Storage System

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Abstract

A system capable of receiving, absorbing, and converting solar energy was designed for use on a satellite in low Earth orbit. The proposed system, an alternative to conventional photovoltaic panels paired with electrochemical batteries, has at the core of its design a latent heat based energy storage system that employs silicon as the phase change material. Thermal to electric conversion is achieved by thermophotovoltaic cells that then provide electrical power for various satellite components. The system was evaluated computationally. Through prediction of the melt and solidification fronts the amount of solar irradiation required to fully utilize the phase change material was determined to be between 4 and 5 kW depending on the orbit. The average temperature of the emitter, used to power the thermophotovoltaic cells, was also predicted throughout an orbit. The emitter temperature range, 1450 to 1850 K, is well-suited for use with commercially available gallium antimony cells.

Keywords

Phase change material, thermal energy storage, thermophotovoltaic cell, radiative heat transfer

1. Introduction

For solar energy to be a viable alternative energy source, an energy storage system is generally required. In a system in which energy can be used in the form of heat (i.e.: space heating for terrestrial applications, or propellant heating for on-orbit applications), a thermally-based energy storage system can offer significant advantages in terms of efficiency, system mass, and cost. State of the art thermal energy storage systems typically use sensible heating; the energy storage and release is accomplished by raising and lowering the temperature of some material. To increase the storage capacity of a sensible heat thermal energy storage system, either the range of operating temperatures or the size of the system must be increased. A latent heat energy storage system primarily stores energy via the reversible melting and solidification of a phase change material; this provides an attractive alternative to sensible heating because it allows relatively constant temperature operation and provides a large energy storage density. A proposed energy storage system that uses silicon as the phase change material and thermophotovoltaic cells for electrical energy conversion was evaluated computationally for use in low Earth orbit.

The phase change material used by a latent heat energy storage system can greatly affect overall performance. Key thermal property requirements of a material used for latent energy storage include: a melting temperature in the desired operating range and a large heat of fusion (for operation primarily on a solid-liquid transition), high specific heat, high density, and high thermal conductivity [1]. The material should also be chemically stable, compatible with container materials, and non-toxic. In addition, the

material should have little to no supercooling and be able to withstand repeated cycling. The material must also be available, abundant, and inexpensive [1, 2, 3], although these requirements may be loosened somewhat for certain niches in which material cost is less of a concern. Current latent heat storage systems typically use paraffin compounds or salt hydrates as the phase change material. These materials do meet many, but not all, of the criteria for an effective phase change material. Paraffin compounds are limited in that they do not transfer heat well and are non-compatible with plastic [2]. Salt hydrates can become unstable and are slightly toxic [3]. Key properties of silicon, suggested for use as a phase change material by Gilpin et al. [4], are compared to paraffin wax and salt hydrates in Table 1. The energy density of silicon is an order of magnitude larger than either of the other two materials and the thermal conductivity at the melting temperature is two orders of magnitude larger. In addition, the phase transition temperature of silicon is well-matched to the band gap of existing thermophotovoltaic cells, allowing use of this phase change material with existing thermal to electric conversion technology [5, 6]. Additionally, the high melting temperature of silicon lends itself to direct thermal energy transfer to a propellant when used as the core of a satellite's propulsion and power systems [4]. The primary focus of this study, however, is on the storage of energy within the silicon throughout an orbit cycle and the interactions with the electrical conversion system.

Table 1
Energy storage material comparison

Material	Energy Density, <i>MJ/kg</i>	Phase Transition Temperature, <i>K</i>	Thermal Conductivity @ <i>T_m</i> , <i>W/mK</i>	Density, <i>kg/m³</i>
Silicon	1.8	1685	20	2330
Paraffin wax	0.189	333	0.180	900
Salt hydrates	0.267	331	0.63	1450

2. Background

The ability to predict melt front progression in the phase change material is necessary when designing a thermal energy storage system. The one-dimensional transient case of conduction as a medium passes through the phase transition temperature and melts or freezes is the Stefan problem, which has been examined since the early 1800s [2,7,8] and solved analytically for a semi-infinite solid with simple boundary conditions in the 1950s [9]. To simulate phase change under more complex conditions and multiple dimensions, numerical modeling is required. Commonly used fixed grid numerical techniques for modeling phase change are the enthalpy method and the temperature transforming method.

The primary fixed grid technique, the enthalpy method, sets the enthalpy of the phase change material equal to the sum of the temperature-dependent sensible and latent heats and uses the same governing equation regardless the phase. This method has been used to successfully model several different problems including melting in one and two-dimensional rectangular, cylindrical, annular, and spherical cases [10 – 15]. Of particular interest to this study are high temperature and high latent heat melting and solidification with radiative boundary conditions. A cylinder of material with a melting temperature of 1050 K and radiative boundary conditions was modeled using a modified enthalpy method and compared to experimental data in [16]. The numerical and experimental results matched well and were used to validate the computational method used by this study.

A second fixed grid technique, the temperature transforming method (TTM), employs the enthalpy-based energy equation and manipulates the thermodynamic properties of the material. When the material is at or near its melting temperature, a range referred to as the mushy zone, the specific heat is increased by the latent heat of fusion divided by the associated mushy zone temperature range. In addition, the viscosity is increased to a very high value in the solid which effectively stops all motion in the solidified portion of the medium [17]. The TTM has been shown to be accurate and effective [18]. Furthermore, the TTM can be employed within existing computational software to incorporate the effects of phase change. Ogoh and Groulx presented a one-dimensional model [19] and a cylindrical finned [20]

model of the phase change process in the commercially available software COMSOL. To simulate phase change they used the temperature transforming method. Their one-dimensional results compared well to an analytical evaluation of the one-dimensional problem.

This computational work evaluates a latent energy storage system using the temperature transforming method. The system was designed for use on a satellite in low Earth orbit. During the sunlight phase of orbit, solar energy is used to melt silicon, the phase change material, while the material is allowed to solidify during eclipse. The energy stored in the phase change material is converted to electricity through the use of thermophotovoltaic cells. The system was modeled in five different low Earth orbits. The melt and solidification fronts were predicted, the solar irradiation required to fully utilize the phase change material was determined, and the temperature of the thermophotovoltaic emitter quantified.

3. Model

A latent energy storage system was designed and evaluated computationally. At the core of the system was a rod of phase change material housed in a cylindrical inner container. A vacuum gap separated the outside of the inner container radially from insulation. Between the inner container and insulation were three radiation shields. The insulation was surrounded by an outer container that housed the entire system. The thermal to electric conversion system, composed of thermophotovoltaic cells, was located above the inner container top surface. Both the inner container and thermophotovoltaic cells were suspended from the insulation via standoffs. A side view of the system is shown in Figure 1 and the system dimensions and materials are detailed in Table 2. Certain aspects of the design were neglected by this study. Specifically, the aperture required for solar input was not included. Obviously, this is a crucial component of the real design, but its exclusion does not affect the analysis and it was therefore neglected for simplicity.

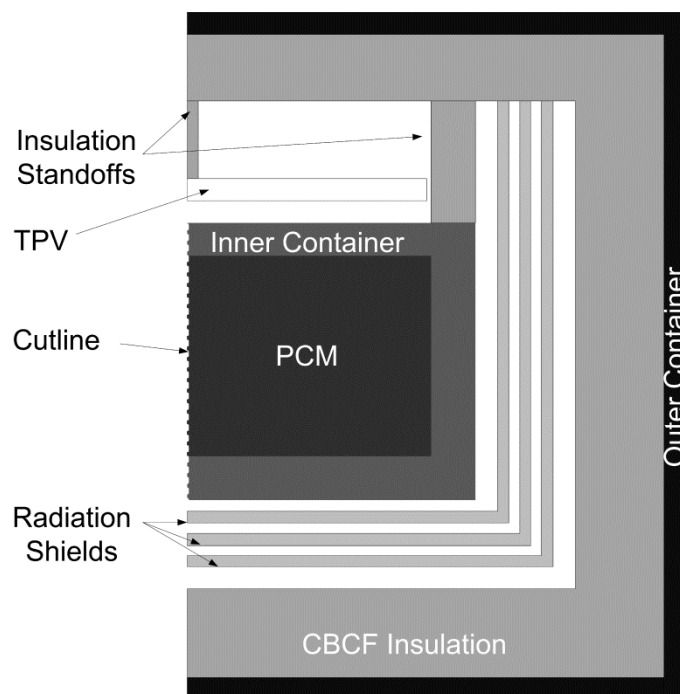


Figure 1: One-half of a cross-sectional view of the full model; the center of the model is on the left, and the axis of symmetry is vertical.

Table 2

System dimensions and materials

Component	Inner Height, <i>m</i>	Inner Radius, <i>m</i>	Outer Height, <i>m</i>	Outer Radius, <i>m</i>	Material
PCM Core	-	-	0.08	0.06	Silicon
Inner Container	0.08	0.06	0.1	0.065	Silicon Carbide
Outer Container	0.295	0.195	0.315	0.205	Graphite
Insulation	0.205	0.115	0.3	0.115	CBCF
Shield 1	0.173	0.077	0.175	0.079	Tantalum
Shield 2	0.183	0.088	0.185	0.09	Tantalum
Shield 3	0.193	0.1	0.195	0.102	Tantalum

A two-dimensional, axisymmetric model, generated in COMSOL Multiphysics 4.2a, was used to evaluate the melting and solidification fronts in the phase change material as well as characterize the system heat transfer. Boundary conditions were selected to accurately represent heat transfer in the space environment. Both steady state and transient studies were performed.

The incoming solar irradiation was represented with a constant surface heat flux applied to the top of the inner container. This value varied, depending on the orbit under analysis. The outer surface of the outer container radiated with an emissivity of 0.1 to an ambient temperature of 300 K for the entire orbit. Although the satellite passes in and out of eclipse, a large solar collector would be attached to the satellite and was assumed to be shading the device. The outer circumference of the inner container, the inner and outer surfaces of the radiation shields, and the inner surface of the insulation were given surface-to-surface radiation conditions and assigned emissivities of 0.8, 0.03, and 0.2, respectively. The radiation exchanged between the container top and the thermophotovoltaic cells was also modeled using surface-to-surface radiation. It was assumed that 50% of the radiation fell below the band gap of the thermophotovoltaic cells and was reflected by a filter; therefore the emissivity of the emitter (inner container top) was set to 0.5. The emissivity of the thermophotovoltaic cells was set to 0.8. A constant temperature boundary condition of 343.15 K was applied to the upper surface of the thermophotovoltaic support to represent the cooling load required to maintain the cells at the desired operating temperature. A volumetric heat sink with magnitude equal to the average electric power produced by the TPV, 170 W, was also applied to the TPV to account for the electric power leaving the system.

It was originally determined that eight tantalum radiation shields with an emissivity of 0.2 would be required, but due to the large aspect ratio of the shield, this proved to be computationally intensive. The eight shields were modeled as three equivalent shields with an emissivity of 0.03, a set-up that was determined to result in the same radiation and conduction losses, but with significantly less computational time.

The temperature transforming method (TTM) was used to model the phase change mechanism. In the TTM, the relevant thermodynamic properties are modified by functions of temperature [7]. The specific heat of the phase change material was modified to account for latent heating, as seen in Eq. 1. First a mushy zone, ΔT_m , a range of temperatures which straddled the melting temperature, was defined. Whenever the silicon temperature was determined to be in this range, its specific heat, $C_{p,m}$, was increased to account for phase change. When the silicon temperature entered the mushy zone a rectangular pulse function, $f(T)$, was used to add the latent heat of fusion, l_f , divided by the mushy zone temperature range, to the specific heat of silicon. The resultant specific heat, $C_p(T)$, incorporates the effects of latent heating, as follows:

$$C_p(T) = f(T) \frac{l_f}{\Delta T_m} + C_{p,m} \quad (1)$$

An iterative approach was used to describe the heat transfer in the system during a particular orbit. Five studies were performed to ensure the process was correctly modeled and convergence was achieved. In the first study, the system was modeled as steady state and a constant temperature condition was applied to the top of the phase change material to determine the temperature profile in the material after a complete melt. This temperature profile was then input into a transient solidification study, study 2, as the initial temperature condition. In study 2, the constant temperature boundary condition was removed and the system was allowed to cool for the time period associated with the eclipse of the orbit under consideration. The final temperature profile of study 2 was then used as an initial condition in study 3. Study 3 was a transient evaluation of the melt occurring during the sunlight phase of orbit under investigation. The irradiation was simulated using a constant power input. The final temperature profile of study 3 served as an initial condition for study 4, and the final temperature profile of study 4 was an input for study 5. Studies 4 and 5 were transient evaluations of the solidification and melt respectively. The results presented are from studies 4 and 5. Five different low Earth orbits were assessed. Altitudes ranging from 400 to 1200 km with a step size of 200 km were evaluated. The time in eclipse and sunlight for each orbit are shown in Table 3.

Table 3
Eclipse and sunlight durations for altitudes

Altitude, <i>km</i>	Eclipse, <i>s</i>	Sunlight, <i>s</i>
400	2000	3500
600	2000	3800
800	2000	4050
1000	1950	4400
1200	1900	4700

Although the system was composed of many different components the phase change material and inner container top results were the most pertinent, therefore the computational mesh in these components was highly refined. The maximum element size was 0.08 cm for the PCM and 0.25 cm for the inner container. A free triangular mesh was used for all components and a total of 30528 elements were used. A direct PARDISO segregated solver was employed by all studies.

4. Validation

The phase change mechanics predicted by the temperature transforming method (TTM) employed in this study were compared to published experimental data as well as an alternative numerical method. The solidification of a silicon cylinder 0.036 m in diameter and 0.02 m in height was modeled. The boundary conditions included an insulated base while the remaining surfaces experienced natural convection and radiation with an ambient room temperature of 300 K. The initial temperature of the cylinder was set to 1175 K and the system was allowed to cool. The temperature history at the center of the cylinder during solidification predicted by the TTM, used by this study, is compared to the experimental and numerical values reported by Elgafy [16] in Figure 2. The TTM simulations match the experimental data more closely than the modified enthalpy method predictions, but share the problem of a slight overshoot at the end of the solidification process. The difference between the experimental and numerical results is most likely due to boundary condition simplifications. For example, the radiative surface was assumed to be diffuse and the emissivity independent of temperature. In addition, the TTM was evaluated for a range of mushy zone sizes. The results for 10 and 20 K mushy zones are shown in Figure 2. The results were insensitive to the size of the mushy zone, therefore a zone size of 10 K was chosen for ease of convergence. Overall, the TTM model accurately matches the experimental data well enough to give confidence in the predicted results.

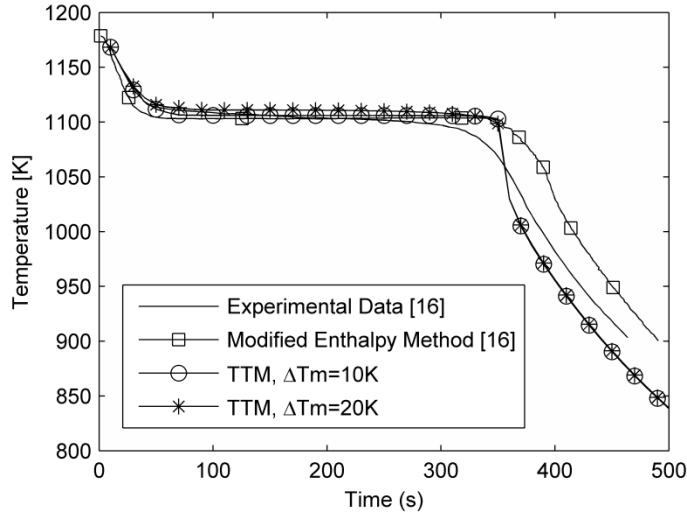


Figure 2: A comparison of experimental and numerical cylindrical solidification temperature histories.

5. Results and Discussion

The phase change material based thermal energy storage system was evaluated at five different orbit altitudes starting at 400 and increasing to 1200 km. The progression of the solidification and melting fronts through the phase change material during each orbit was modeled and used to determine solar irradiation required to fully utilize the melt. The results from the 600 km orbit will be discussed in detail, as they are representative of the results seen at all five orbits, and then a summary of all results will be provided.

The temperature along the centerline of the phase change material and inner container, shown as a dotted line in Figure 1 and referred to as the cutline, is shown for discharge of the system as a function of time at an altitude of 600 km in Figs. 3 and 4. The temperature from the inner container bottom ($z = 10$ to 11 cm) through the phase change material ($z = 11$ to 19.4 cm) to the inner container top ($z = 19.4$ to 20.4 cm) during the first 200 s of solidification is shown in Figure 3. Initially, the top of the container and phase change material are above 1800 K. This high temperature results in significant radiative losses, causing both the container top and the upper portion of the phase change material to cool rapidly with the temperature at the top of the phase change material dropping 155 K in the first 40 s. Over the next 160 s, the rate at which the temperature drops is slower due to lower average temperatures and the solidification of a small portion of the phase change material.

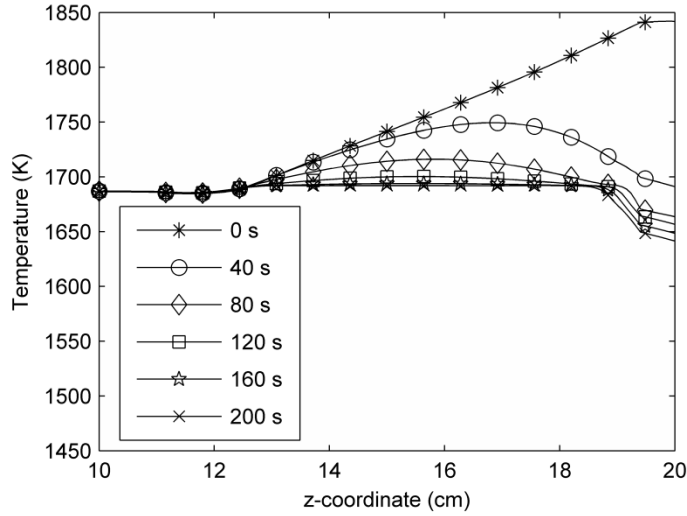


Figure 3: The temperature history along the cutline during the first 200 s of discharge.

The temperature as a function of time along the cutline during the entire eclipse is shown in Figure 4. At the start of discharge, $t = 0$ s, the phase change material, from $z = 12.5$ cm to $z = 19.4$ cm, is considered fully melted. For this study, when more than 80% of the phase change material volume is at a temperature greater than the melting temperature of 1687 K, the material is referred to as fully melted. The solidification front moves downward at an average rate of 0.0025 cm/s and the majority of the core is solidified after 2000 s. The lack of radiation shielding and conduction paths created by the insulation standoffs above the inner container top make this region above the phase change material prone to more significant thermal losses. These large radiative and conductive losses from the inner container top drive the solidification front. In addition, it can be seen that the heat transfer rate from the container top ($z = 19.4$ to 20.4 cm) is higher than that from the PCM. This is due to its much larger thermal diffusivity of the container material.

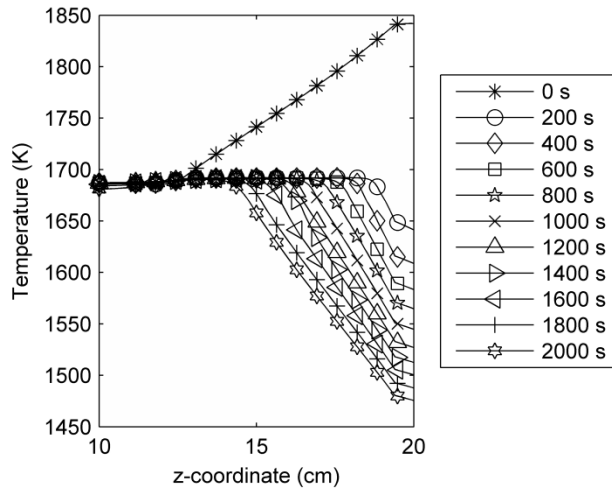


Figure 4: The temperature history along the cutline during the discharge of the system at 600 km.

The temperature as a function of time along the cutline during the entire daylight phase of the orbit is shown in Figure 5. The temperature profile along the cutline at the end of eclipse (Figure 4, $t = 2000$ s) serves as the initial condition for the system as it enters the daylight phase of orbit. At the charge start, $t = 0$ s, the top of the container and phase change material are at 1485 K. In less than 400 s, the top of the phase change material ($z = 19.4$ cm) is at 1751 K, 64 K above the melting point of silicon. The melt front moves downward at an average rate of 0.0013 cm/s. The core is considered fully melted after 3800

s. Temperatures in the top portion of the phase change material exceed the melting temperature for the majority of the charge resulting in significant sensible heating. The power required to ensure a full melt for the 600 km orbit was determined iteratively to be 4.4 kW.

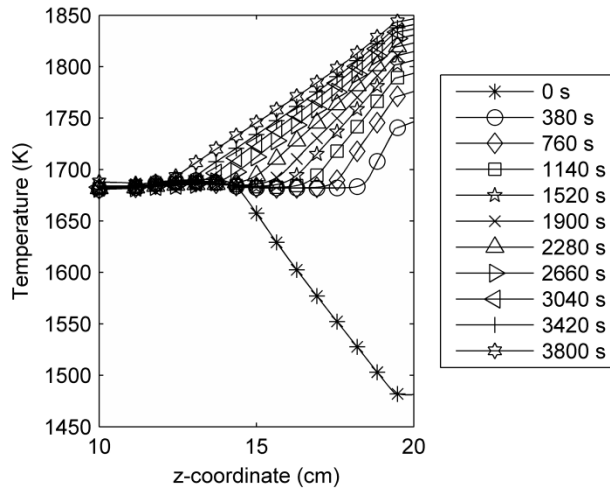


Figure 5: The temperature history along the cutline during the charge of the system at 600 km.

The progression of the solidification front during the 600 km orbit discharge is shown in Figure 6. Each shade is an isotherm associated with the melting temperature. The solidification front moves inward with time. The thermophotovoltaic cells are located above the inner container top, preventing radiation shielding from being used in that area, therefore more heat is lost from the top of the system than the sides or bottom resulting in faster downward front motion.

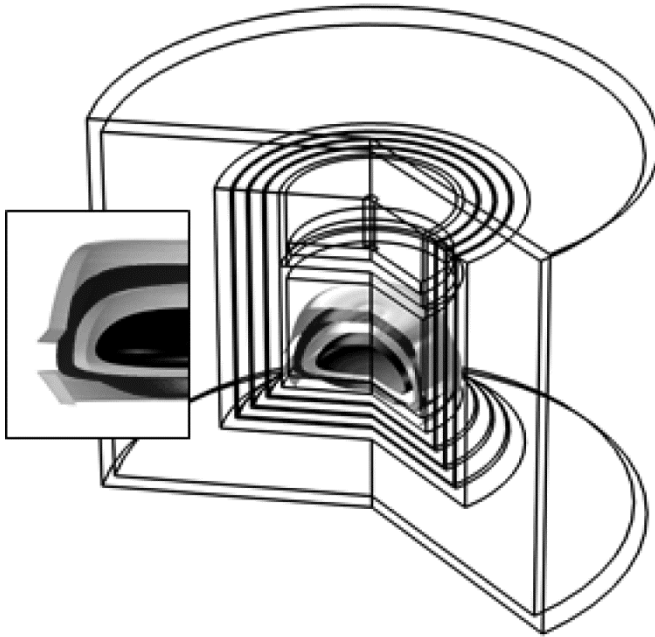


Figure 6: The solidification front during discharge of the system at 600 km.

The progression of the melt front during the 600 km orbit discharge is shown in Figure 7. Again, each shade is an isotherm associated with the melting temperature. The melt front moves down from the inner container top with time. The front moves more quickly along the edges of the container due to the

higher thermal diffusivity of the container material. Towards the bottom of the container there are two elliptically shaped isotherms that correlate to an unsolidified region that existed at the start of the charge.

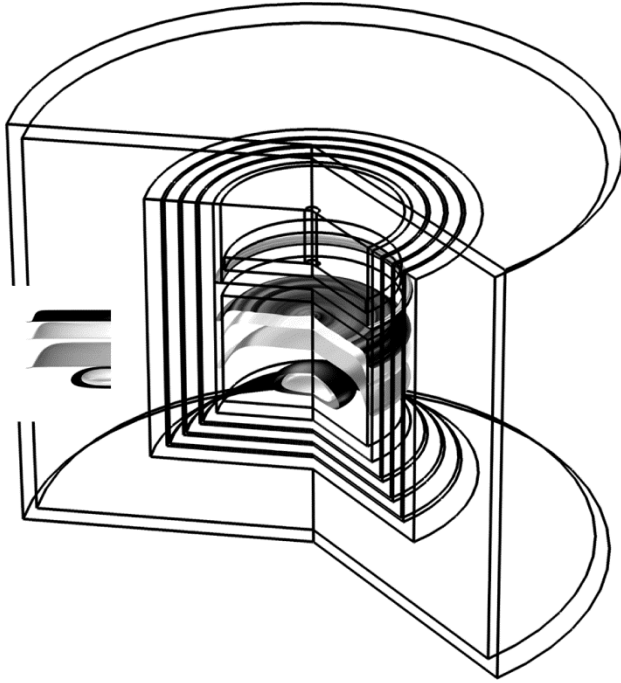


Figure 7: The melt front during charge of the system at 600 km.

The inner container top acts as an emitter for the thermophotovoltaic cells. The average temperature of the emitter, as a function of time, for discharge and charge is shown in Figure 8. The temperatures experienced by the emitter during orbit range from 1450 to 1850 K. This temperature range promotes pairing with gallium antimony (GaSb) thermophotovoltaic cells, which have a band gap of 0.72 eV, for thermal to electric conversion. The emitter temperature can be directly correlated to thermophotovoltaic power output.

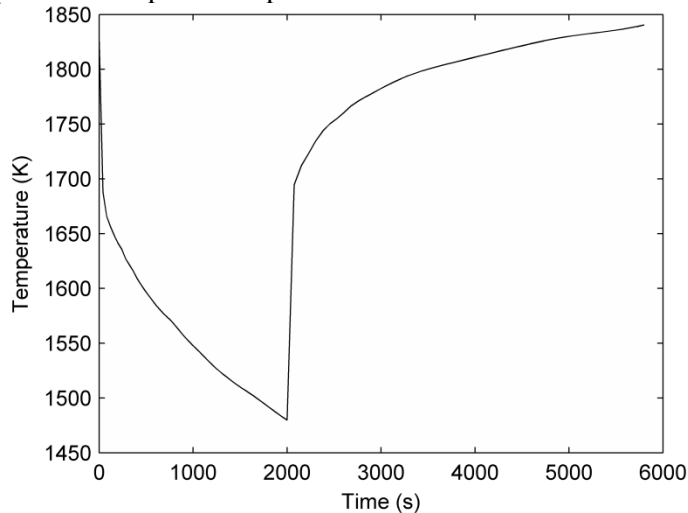


Figure 8: Average temperature of the emitter as a function of time.

The latent thermal energy storage system was analyzed in five different low Earth orbits to determine the solar irradiation required to fully melt a fixed sized system. Full melt was defined as 80% of the phase change material volume being at or above the melt temperature. The required concentrated solar heat fluxes are 4.6, 4.4, 4.3, 4.15, and 4.0 kW for 400, 600, 800, 1000, and 1200 km orbits respectively.

6. Conclusions

The melt and solidification fronts in the phase change material of a thermal energy storage system designed for use on a satellite in low Earth orbit were predicted. The amount of solar irradiation required to fully utilize the phase change material of silicon was determined to be between 4 and 5 kW depending on the orbit. The shapes of the fronts were predicted and found to be dictated by system design, in particular by the system radiation shielding and insulation schemes. The average temperature of the emitter, used to power thermophotovoltaic cells, throughout orbit was also predicted. The emitter temperatures ranged from 1450 to 1850 K, making thermal electric conversion via commercially available gallium antimony cells feasible. Further work is required to fully vet this thermal electrical conversion pairing. In addition, the melt and solidification front predictions must be used to develop an optimized design.

Acknowledgments

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References

- [1] M. Liu, W. Saman, and F. Bruno, Review on storage materials and thermal performance enhancement techniques for high temperature phase change thermal storage systems, *Renewable and Sustainable Energy Reviews* 16 (2012) 2118-2132.
- [2] A. Sharma, V.V. Tyagi, C.R. Chen, and D. Buddhi, Review on thermal energy storage and phase change materials and applications, *Renewable and Sustainable Energy Reviews* 13 (2009) 318 – 345.
- [3] M. Kenisarin and K. Mahkamov, Solar energy storage using phase change materials, *Renewable and Sustainable Energy Reviews* 11 (2007) 1913 – 1965.
- [4] M.R. Gilpin, D.B. Scharfe, M.P. Young, and A.P. Pancotti, Molten boron phase-change thermal energy storage to augment solar thermal propulsion systems, In: 47th AIAA/ASME/SAE/ASEE Joint Propulsion Conference Proceedings, 2011.
- [5] W. Chan, R. Huang, C. Wang, J. Kassakian, J. Joannopoulos, and I. Celanovic, Modeling low-bandgap thermophotovoltaic diodes for high-efficiency portable power generators, *Solar Energy Materials and Solar Cells* 94 (2010) 509 – 514.
- [6] L.G. Ferguson and L.M. Fraas, Theoretical study of GaSb PV cell efficiency as a function of temperature, *Solar Energy Materials and Solar Cells* 39 (1995) 11 – 18.
- [7] Y. Dutil, D. Rousse, N. Salah, S. Lassue, and L. Zalewski, A review of phase-change materials: mathematical modeling and simulations, *Renewable and Sustainable Energy Reviews* 15 (2011) 112 – 130.
- [8] M. Medrano, A. Gil, I. Martorell, X. Potau, and L. Cabeza, State of the art on high-temperature thermal energy storage for power generation, Part 2 – Case studies, *Renewable and Sustainable Energy Reviews* 14 (2010) 56 – 72.
- [9] G. W. Evans, A note on the existence of a solution to a problem of Stefan, *Quarterly of Applied Mathematics* 9 (1951) 185 – 193.
- [10] C. Swaminathan and V. Voller, On the enthalpy method, *International Journal of Numerical Methods in Heat and Fluid Flow*, 3, (1993), pp. 233-244.
- [11] V. Voller, M. Cross, and N. Markatos, An enthalpy method for convection/diffusion phase change, *International Journal of Numerical Methods in Engineering* 24 (1987) 271 – 284.
- [12] V. Alexiades and A. Solomon, *Mathematical modeling of melting and freezing processes*, Hemisphere Publishing, USA, 1993.
- [13] B. Nedjar, An enthalpy-based finite element for nonlinear heat problems involving phase change, *Computational Structures* 80 (2002) 9 – 21.
- [14] A. Date, Novel strongly implicit enthalpy formulation for multidimensional Stefan problems, *Numerical Heat Transfer B* 21 (1992) 231 – 251.

- [15] J. Caldwell and C. Chan, Numerical solutions of Stefan problem in annuli by enthalpy method and heat balance integral method, *Communications in Numerical Methods in Engineering* 17 (2001) 395 – 405.
- [16] A. Elgafy and O. Mesalhy, Numerical and experimental investigations of melting and solidification processes of high melting point PCM in a cylindrical enclosure, *Journal of Heat Transfer*, 126 (2004) 869 – 875.
- [17] D.K. Gartling, Finite element analysis of convective heat transfer problems with change of phase, in: *Computer Methods in Fluids*, 1980, 257 – 284.
- [18] Y. Cao and A. Faghri, A numerical analysis of phase change problem including natural convection, *Journal of Heat Transfer* 112 (1990) 812 – 815.
- [19] W. Ogoh and D. Groulx, Stefan's problem: Validation of a one-dimensional solid-liquid phase change heat transfer process, In: *Proceedings of the COMSOL Conference*, 2010.
- [20] D. Groulx and W. Ogoh, Solid-liquid phase change simulation applied to a cylindrical latent heat energy storage system, In: *Proceedings of the COMSOL Conference*, 2009.